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# A Simple Method for the Design of Optimal Damper Configurations in MDOF Structures

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Existing methods for the design of optimal configurations of supplemental dampers are usually not simple enough to be used routinely and typically lead to different damper sizes at virtually every story. One exception is the Sequential Search Algorithm, which lets the designer control the number of different damper sizes. In this paper, a simplification to the Sequential Search Algorithm is proposed so that the resulting procedure, designated as Simplified Sequential Search Algorithm, can be easily integrated into conventional design procedures used by practicing engineers dealing with damper-added structures. In the case of linear viscous dampers, it was found that the efficiency of damper configurations given by the proposed Simplified Sequential Search Algorithm is comparable to the efficiency of damper configurations given by more sophisticated procedures. The applicability of the method is limited to those cases where the response of the structure with added dampers remains linear.

## INTRODUCTION

The seismic response of structures subjected to earthquake excitations may be effectively reduced by incorporating any of various kinds of available passive energy dissipation devices (Soong and Dargush 1997). Such devices, commonly referred to as dampers, can be added to either newly designed or existing structures and have already been implemented in a number of real structures for control of earthquake-induced vibrations (Constantinou et al. 1998).

As for any other structural component, an efficient use of supplemental dampers may result in considerable cost savings. For instance, in order to achieve a given performance objective, the number of optimally placed dampers of equal size that must be added may be significantly less than the needed number of the same dampers non-optimally located. Surprisingly, despite the important benefits that may be obtained, little work has been done in order to improve the effectiveness of damper configurations. A significant amount of experimental and analytical research has been done regarding the applicability of passive energy dissipation devices for structural control, but the development of efficient procedures leading to optimal damper arrangements has received less attention.

The ideal method for the design of optimal configurations of dampers that are to be incorporated into a structure should be both practical and efficient. It should be practical in the sense that it is simple enough to be used routinely by practicing engineers. It should also be practical in the sense that it is capable of controlling the number of different damper sizes

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to be used, which is desirable and convenient. Finally, it should be efficient in the sense that the resulting damper configuration (i.e., size and location of added dampers) minimizes the total amount of added damping necessary to reach a given performance objective. While a few efficient design procedures have been developed and can be found in the literature, they are typically not sufficiently practical as defined above.

In this paper, a simplification of an existing design methodology is proposed so that the resulting procedure satisfies all the desirable characteristics mentioned in the former paragraph. Complete details are presented for the special case of linear viscous dampers. Applicability of the proposed method to other kinds of devices is then discussed.

## THE SEQUENTIAL SEARCH ALGORITHM (SSA)

Based on the controllability index method presented by Cheng and Pantelides (1988), Zhang and Soong (1992) proposed a sequential procedure to find the optimal placement of viscoelastic dampers. The procedure can be conceptually described as follows. The seismic random response of the bare structure (i.e., the structure without any added dampers) is obtained using the transfer matrix method. Mean square values of interstory drifts are calculated and used as optimal location indices. The greatest optimal location index indicates the optimal placement of the first damper. The stiffness and damping of the added damper are incorporated into the mathematical model of the structure and new optimal location indices are calculated. The second damper is then placed at the story where the revised optimal location index is a maximum. The procedure is repeated until all necessary dampers have been placed in the structure. All dampers are assumed to have the same size. The modal strain energy method is used to develop simple equations relating the number and size of the dampers and the equivalent damping ratio. In this paper, this method will subsequently be referred to as the Sequential Search Algorithm (SSA). Its efficiency has been recently validated further by Shukla and Datta (1999).

Since all dampers have the same size, the SSA is more practical than other optimization methods, which usually lead to a different damper size at each story. Because equations relating the number and size of the dampers are provided, the SSA is very flexible in that it gives the designer a number of possible choices. For instance, if the damper size is constrained, then the number of dampers can be adjusted. If, for other reasons, the number of dampers is subjected to limitations, then the damper size can be conveniently selected. Moreover, the SSA is not limited to damper configurations of equally sized dampers. The effect of two equal dampers placed at the same story is equivalent to the effect of one damper whose size is twice the size of the original dampers and placed at the same story. Therefore, by limiting the number of dampers that can be placed at the same story, the number of different damper sizes can be controlled.

Despite the advantages mentioned above, the required mathematical computations may be sufficiently complex to discourage usual application of the SSA by practicing engineers. In an attempt to overcome this shortcoming, a simplification is presented in the next section.

## PROPOSED SIMPLIFIED SEQUENTIAL SEARCH ALGORITHM (SSSA)

The essential idea behind the SSA is simple: dampers are placed sequentially where their effect is maximized. In each step, the properties of the already added dampers are taken into account. The only complexity arises from the way optimal location indices are computed. If these indices could be calculated using procedures similar to those usually applied in structural design, the resulting method will be simple enough to be used habitually while retaining all the advantages of the SSA mentioned before.

In accordance to the observations made in the former paragraph, the following simplification to the SSA is proposed. Optimal location indices are given simply by:

$$\gamma_i = \alpha_1 \delta_i + \alpha_2 \dot{\delta}_i \quad (1)$$

where $\gamma_i$ is the optimal location index for the i-th story, $\alpha_1$ and $\alpha_2$ are constant coefficients and $\delta_i$ and $\dot{\delta}_i$ are interstory drift and velocity at the i-th story, which in turn are calculated deterministically for any ground acceleration history compatible with the expected seismic events at the site. Constants $\alpha_1$ and $\alpha_2$ are defined according to whether the energy dissipation characteristics of the dampers to be used are displacement-dependent, velocity-dependent or both. With optimal location indices calculated as indicated by Equation 1, dampers are then placed sequentially as explained before, i.e., where the optimal location index is a maximum. In this paper, this approach will subsequently be referred to as the Simplified Sequential Search Algorithm (SSSA).

For the particular case of linear viscous dampers, which have been proved to be successful in reducing the seismic response of structures (Constantinou and Symans 1992, Reinhorn et al. 1995, Seleemah and Constantinou 1997), the SSSA specializes as follows. Because viscous dampers are essentially velocity-dependent devices, their energy dissipation capabilities are fully exploited where the interstory velocity is a maximum. Therefore, $\alpha_1 = 0$ and $\alpha_2 = 1$, i.e., the optimal location index is given simply by the interstory velocity. The damper size, which in this case is the viscous coefficient $c$, may be estimated using the strain energy method (same approach as in Zhang and Soong 1992). The energy dissipated by a single damper at the i-th story over a harmonic cycle at the fundamental frequency is given by:

$$E_{Di} = \pi c \frac{2\pi}{T} \delta_i^2 \cos^2 \theta_i = \frac{2\pi^2 c \delta_i^2 \cos^2 \theta_i}{T} \quad (2)$$

where $c$ is the viscous coefficient, $T$ is the fundamental period of the structure, $\delta_i$ is the interstory drift at the i-th story and $\theta_i$ is the inclination angle of the damper placed at the i-th story. Assuming that all stories have the same height and that the first modal shape is a straight line, interstory drifts are given by:

$$\delta_i = \frac{1}{n} \quad (3)$$

where $n$ is the number of stories. The total energy dissipated by the dampers is then given by:

$$E_D = \sum_{i=1}^{n_d} E_{Di} = \sum_{i=1}^{n_d} \frac{2\pi^2 c \cos^2 \theta}{T n^2} = \frac{2\pi^2 c\, n_d \cos^2 \theta}{T n^2} \quad (4)$$

where $n_d$ is the number of dampers. It must be noted that inclination angles $\theta_i$ are assumed to be equal at all stories, which is consistent with the assumption that all stories have the same height. Assuming that viscous dampers add no stiffness to the structure, the total strain energy of the system is given by:

$$E_S = \sum_{i=1}^{n} \frac{1}{2} K_i \, \delta_i^2 = \frac{1}{2 n^2} \sum_{i=1}^{n} K_i \tag{5}$$

where $K_i$ is the lateral stiffness of the i-th story. The equivalent damping ratio due to the action of the added dampers is then given by:

$$\xi_d = \frac{E_D}{4 \pi E_S} = \frac{\pi c \, n_d \cos^2 \theta}{T \sum_{i=1}^{n} K_i} \tag{6}$$

from which:

$$c = \frac{\xi_d \, T \sum_{i=1}^{n} K_i}{\pi \, n_d \cos^2 \theta} \tag{7}$$

Equation 7 gives the damper size $c$ as a function of the number of dampers $n_d$, the equivalent damping ratio $\xi_d$ and characteristic quantities of the structure. It must be noted that $\xi_d$ is the equivalent damping ratio due to the added dampers only. The total equivalent damping ratio of the structure with added dampers $\xi_T$ is $\xi_d$ plus the inherent damping ratio of the structure $\xi_o$, i.e.:

$$\xi_T = \xi_o + \xi_d \tag{8}$$

Despite its simplicity, Equation 7 is a powerful design tool in that, for a given structure (defined by $T$ and $\sum_{i=1}^{n} K_i$), a given damper inclination (defined by $\theta$) and a given performance objective (defined by $\xi_d$), there are a number of combinations of the remaining variables ($c$ and $n_d$) from which the designer can choose at convenience.

## EVALUATION OF THE EFFICIENCY OF THE SSSA

The proposed SSSA is a modification of the SSA so that the necessary calculations do not require any expertise other than that necessary to perform customary analysis of damper-added structures. However, simplicity is achieved at the expense of a somewhat less rigorous basis. Therefore, there is still a question about the efficiency of the proposed methodology. In what follows, the efficiency of the SSSA for the particular case of linear viscous dampers is evaluated by comparing the response of structures with added dampers designed using the SSSA with the response of the same structures with added dampers designed in accordance with more sophisticated techniques.

### COMPARISON WITH OPTIMAL DESIGN USING OPTIMAL CONTROL THEORY

Gluck et al. (1996) adapted Optimal Control Theory (OCT) using a linear quadratic regulator to design linear passive viscous or viscoelastic devices according to their deformations and velocities. The efficiency of the method is given by the fact that the response of a structure with added dampers designed by this procedure is essentially equivalent to the response of the same structure actively controlled, as long as the response of the structure is dominated by a single mode of vibration.

The following example is taken from Ribakov and Gluck (1999). The parameters of a seven-story shear building with stiff beams are summarized in Figure 1 (left). Optimal damping coefficients calculated by Ribakov and Gluck (1999) using OCT are shown in Figure 1 (right). The inherent damping ratio of the structure $\xi_o$ is assumed as 1%.

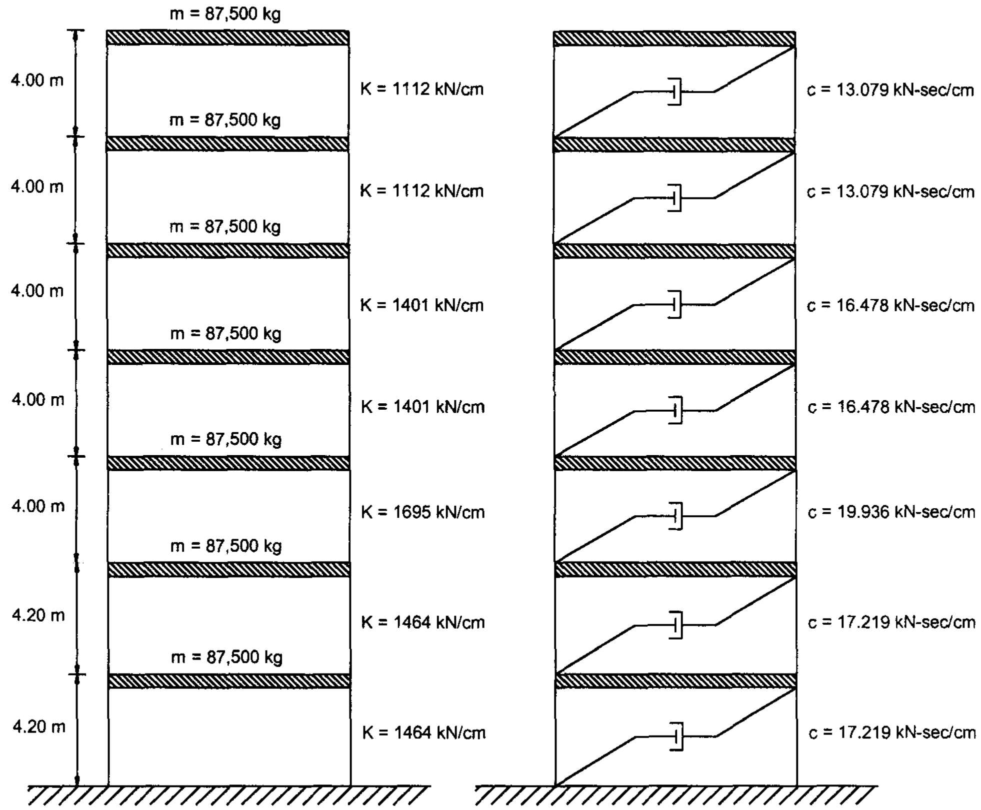


**Figure 1.** (left) Structural properties of the uncontrolled seven-story structure. (right) Damping coefficients calculated by Ribakov and Gluck (1999) using Optimal Control Theory.

Numerical simulations were performed using the commercially available computer program SAP-2000 (Computers and Structures Inc. 2000). The following ground acceleration time histories were used as input excitations: El Centro S00E (scaled to PGA = 0.369 g), Kobe EW (scaled to PGA = 0.184 g), Taft N21E (scaled to PGA = 0.321 g) and the Rinaldi 318 record of Northridge earthquake (scaled to PGA = 0.297 g). The records were scaled in such a way that the maximum interstory drift of the structure with optimal dampers as given by OCT is equal to 0.50%, which ensures elastic behavior.

Next, the proposed SSSA is applied. For each ground motion, the response of the bare structure is obtained for increasing values of the damping ratio until maximum interstory drifts are equal to those obtained for the optimally damped structure using OCT. The resulting damping ratios are then equivalent total damping ratios $\xi_T$. In other words, the performance objective of the SSSA is the response level that is achieved through OCT, which makes possible an appropriate comparison in terms of efficiency. From Equation 8, the equivalent damping ratio that must be added through dampers is then:

$$\xi_d = \xi_T - \xi_o \tag{9}$$

As mentioned before, the SSSA may lead to several possible damper arrangements. In this study, only two strategies are considered. The first strategy will be subsequently referred to as no-story repetition, which allows no more than one damper per story. This is achieved by calculating optimal location indices only for those stories where no dampers have been placed. Clearly, this strategy is suitable only if the number of dampers to be added is less than the number of stories, i.e., $n_d < n$. When $n_d = n$ (= 7 in this example), there are equal dampers at all stories and no placement procedure is needed. Although this case is not an application of the SSSA, it is nevertheless considered for comparison purposes and it will be referred to as uniform distribution. The second strategy will be subsequently referred to as story repetition, which allows as many dampers per story as indicated by the SSSA. The following values of $n_d$ are considered: for the no-story repetition strategy, $n_d$ = 5 and 6; for the story repetition strategy, $n_d$ = 5, 6, 7, 8, and 9.

With the values of $\xi_d$ given by Equation 9 and the values of $n_d$ mentioned in the former paragraph, damping coefficients $c$ were obtained using Equation 7, where $T$ = 0.7354 sec, $\sum_{i=1}^{n} K_i$ = 9649 kN/cm and $\theta = 0°$. Resulting damper sizes are summarized in Table 1.

**Table 1.** Application of the SSSA: damping ratios and damper coefficients for the 7-story structure

| | | El Centro | Kobe | Taft | Rinaldi |
|---|---|---|---|---|---|
| $\xi_T$ | | 4.8 % | 5.9 % | 5.7 % | 6.8 % |
| $\xi_o$ | | 1 % | 1 % | 1 % | 1 % |
| $\xi_d$ | | 3.8 % | 4.9 % | 4.7 % | 5.8 % |
| $c$ [kN-sec/cm] | $n_d = 5$ | 17.166 | 22.135 | 21.232 | 26.201 |
| | $n_d = 6$ | 14.305 | 18.446 | 17.693 | 21.834 |
| | $n_d = 7$ | 12.261 | 15.811 | 15.165 | 18.715 |
| | $n_d = 8$ | 10.729 | 13.834 | 13.270 | 16.375 |
| | $n_d = 9$ | 9.537 | 12.297 | 11.795 | 14.556 |

For both strategies, dampers are placed as mentioned before, i.e., sequentially and as indicated by maximum interstory velocities. All damper locations are summarized in Table 2, where it can be seen that, for a given placement strategy, resulting damper locations are very similar to each other.

It is clear that a meaningful comparison of the efficiency of the damper configurations given by OCT and the SSSA should take into account both the displacement response and the amount of added damping. In this study, the displacement response is represented by the maximum interstory drift, which is a useful indicator of the displacement demand imposed on the structure. The amount of added damping is represented by the sum of the damping coefficients of all added dampers, i.e.:

$$C = \sum_{i=1}^{n_d} c_i \tag{10}$$

Figure 2 shows maximum interstory drift vs. total added damping $C$ for all of the ground motions and damper configurations mentioned before.

**Table 2.** Application of the SSSA: damper locations for the seven-story structure

| | $n_d$ | El Centro | Kobe | Taft | Rinaldi |
|---|---|---|---|---|---|
| no-story repetition | 5 | 1-2-3-4-6 | 1-2-3-4-5 | 1-2-3-4-6 | 1-2-3-4-6 |
| | 6 | 1-2-3-4-5-6 | 1-2-3-4-5-6 | 1-2-3-4-5-6 | 1-2-3-4-5-6 |
| story repetition | 5 | 1-1-1-2-2 | 1-1-1-2-2 | 1-1-1-2-2 | 1-1-2-2-4 |
| | 6 | 1-1-1-1-2-2 | 1-1-1-1-2-2 | 1-1-1-2-2-2 | 1-1-1-2-2-4 |
| | 7 | 1-1-1-1-1-2-2 | 1-1-1-1-2-2-2 | 1-1-1-1-2-2-2 | 1-1-1-2-2-2-4 |
| | 8 | 1-1-1-1-1-2-2-2 | 1-1-1-1-1-2-2-2 | 1-1-1-1-2-2-2-2 | 1-1-1-1-2-2-2-4 |
| | 9 | 1-1-1-1-1-1-2-2-2 | 1-1-1-1-1-2-2-2-2 | 1-1-1-1-1-2-2-2-2 | 1-1-1-1-2-2-2-4-4 |

If the sequential approach indeed optimizes damper locations, then the story repetition strategy should be more efficient than the no-story repetition strategy. This is confirmed by the results shown in Figure 2, where it can be seen that maximum interstory drifts for the story repetition strategy are always less than those for the no-story repetition strategy. In turn, the no-story repetition strategy should be more efficient than the uniform distribution, because in this latter case there is no optimization of damper locations. This is also confirmed by the results shown in Figure 2, where maximum interstory drifts for the no-story repetition strategy are always less than those for the uniform distribution.

It can also be observed in Figure 2 that maximum interstory drifts corresponding to the story repetition strategy are always less than those obtained using the damper configuration given by OCT. They are achieved with a significantly less amount of total added damping $C$ for El Centro earthquake, slightly less amount of $C$ for Kobe and Taft earthquakes, but at the expense of more total damping for the Rinaldi ground excitation.

In summary, it can be inferred that the sequential approach indeed optimizes damper location and that the efficiency of damper configurations given by the SSSA is similar to the efficiency of the damper configuration given by OCT.

## COMPARISON WITH OPTIMAL DESIGN FOR MINIMUM TRANSFER FUNCTIONS

Takewaki (1997) presented a procedure to find the optimal damper configuration so that the resulting sum of amplitudes of transfer functions of interstory drifts, evaluated at the undamped fundamental natural frequency of the structure, is minimized. The procedure is subjected to a constraint on the sum of the damping coefficients of the added dampers, i.e., the quantity $C$ as defined by Equation 10. For a given value of $C$, the algorithm finds the optimal distribution of damping coefficients $c_i$ along the stories of the structure. The algorithm is powerful enough to indicate $c_i = 0$ at stories where no dampers are needed. The efficiency of the procedure comes from the very definition of the optimization criterion.

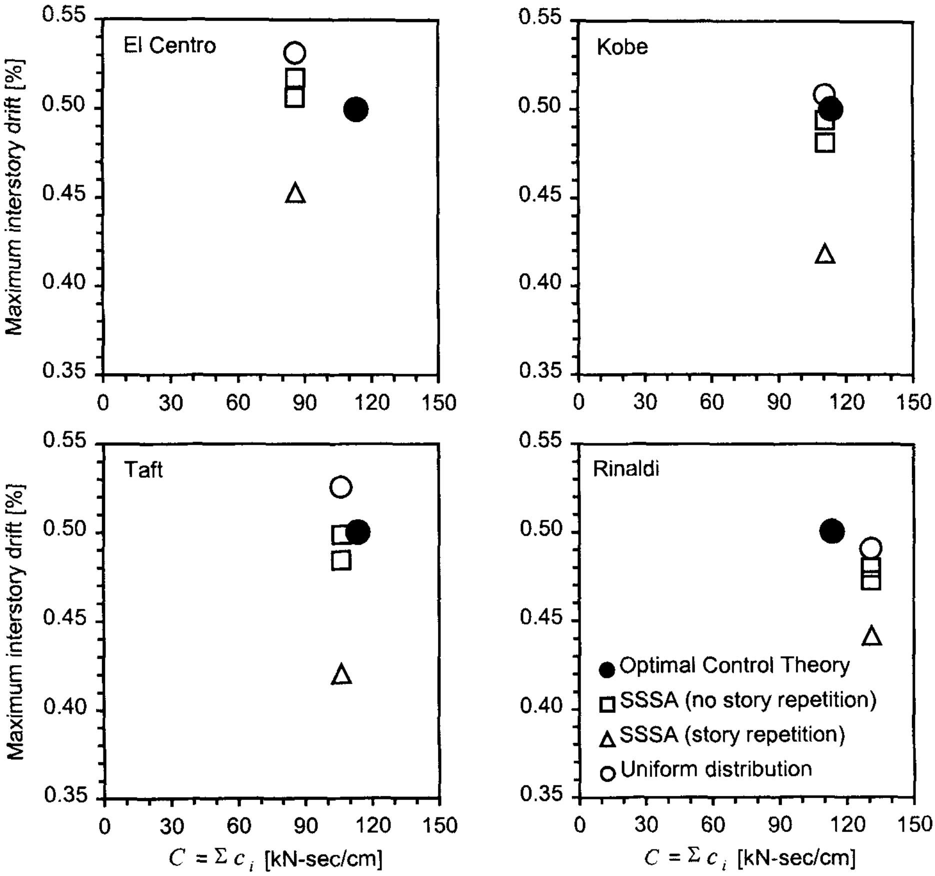


**Figure 2.** Comparison between the SSSA and Optimal Control Theory: maximum interstory drift vs. total added damping $C = \Sigma c_i$ for the seven-story structure.

The following example is taken from Takewaki (1997). The parameters of a six-story shear building with stiff beams are shown in Figure 3 (left). Optimal damping coefficients for minimum transfer functions calculated by Takewaki (1997) are shown in Figure 3 (right). The inherent damping of the structure $\xi_o$ is neglected. Ground acceleration histories used as input excitations in the numerical simulations are the same ground motions mentioned before and scaled to the same PGAs. Again, program SAP-2000 was used.

Takewaki's approach leads to an optimal distribution of a total damping $C$, but does not include any guidelines to calculate $C$ itself. Consequently, an appropriate comparison between the SSSA and optimal damper placement for minimum transfer functions should be made in terms only of the *distribution* of $C$. In order to do so, damping coefficients $c$ for the SSSA are calculated simply by dividing the total added damping $C$ (= 90 kN-sec/cm in Takewaki's example) by the number of dampers $n_d$, i.e.:

$$c = \frac{C}{n_d} \tag{11}$$

The SSSA is then applied as follows. For a given $n_d$, the damper size is given by Equation 11 using $C$ = 90 kN-sec/cm. Damper placement strategies considered in this case are the same strategies mentioned before. For the no-story repetition strategy, $n_d$ = 4 and 5. For the story repetition strategy, $n_d$ = 4, 5, 6, 7 and 8. Resulting damper sizes are summarized in Table 3.

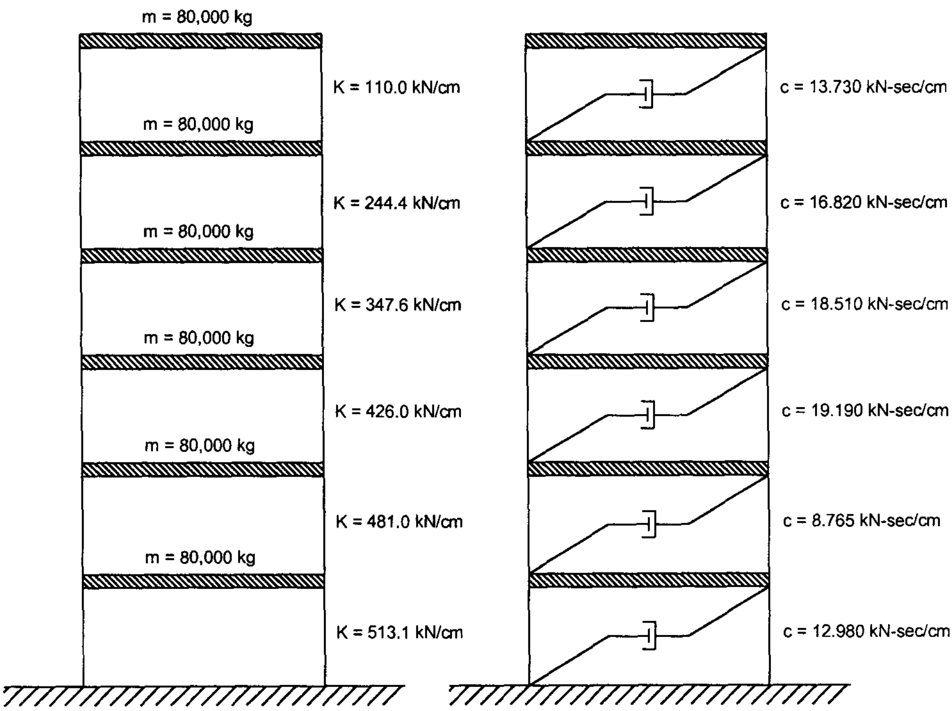


**Figure 3.** (left) Structural properties of the uncontrolled six-story structure. (right) Optimal damping coefficients for minimum transfer functions calculated by Takewaki (1997).

**Table 3.** Application of the SSSA: Damper locations for the six-story structure

| $n_d$ | $c$ [kN-sec/cm] | El Centro | Kobe | Taft | Rinaldi |
|---|---|---|---|---|---|
| 4 | 22.50 | 2-4-5-6 | 3-4-5-6 | 3-4-5-6 | 3-4-5-6 |
| 5 | 18.00 | 2-3-4-5-6 | 2-3-4-5-6 | 1-2-4-5-6 | 1-3-4-5-6 |
| 6 | 15.00 | 2-3-4-5-5-6 | 3-4-4-5-5-6 | 1-1-2-4-5-6 | 1-1-3-4-5-6 |
| 7 | 12.86 | 2-3-4-4-5-5-6 | 3-4-4-5-5-6-6 | 1-1-2-3-4-5-6 | 1-1-2-3-4-5-6 |
| 8 | 11.25 | 2-3-4-4-5-5-6-6 | 2-3-4-4-5-5-6-6 | 1-1-2-2-3-4-5-6 | 1-1-2-2-3-4-5-6 |

For both strategies, dampers are placed as previously explained, i.e., sequentially and as indicated by maximum interstory velocities. In Takewaki's example, the inherent damping of the structure is neglected, hence $\xi_o$ = 0 was assumed for the bare structure. All obtained damper locations are shown in Table 3. There is no distinction between the no-story repetition and story repetition strategies because both led to the same damper locations for $n_d$ = 4 and 5. Damper locations are very similar for $n_d$ = 4 and somewhat different for greater values of $n_d$, but in all cases there are no more than two dampers at a given story. In other words, dampers do not concentrate at particular locations.

For the reasons explained above, the total added damping $C$ is the same for both the SSSA and optimal placement for minimum transfer functions. Therefore, the efficiency of the corresponding damper configurations is evaluated in terms of the displacement response only. Since Takewaki's approach does not aim to minimize the maximum interstory drift but the *sum* of all interstory drifts, the displacement response is represented in this case by the sum of interstory drifts. All results are shown in Figure 4, where it can be seen that the sum of interstory drifts for the damper configurations given by the SSSA is always either essentially equal to or slightly greater than that corresponding to the damper configuration given by the minimum transfer functions approach, and they are achieved with the same total added damping $C$. Thus, it can be concluded that the efficiency of the damper configurations given by the SSSA is similar to the efficiency of the damper configuration given by optimal placement for minimum transfer functions.

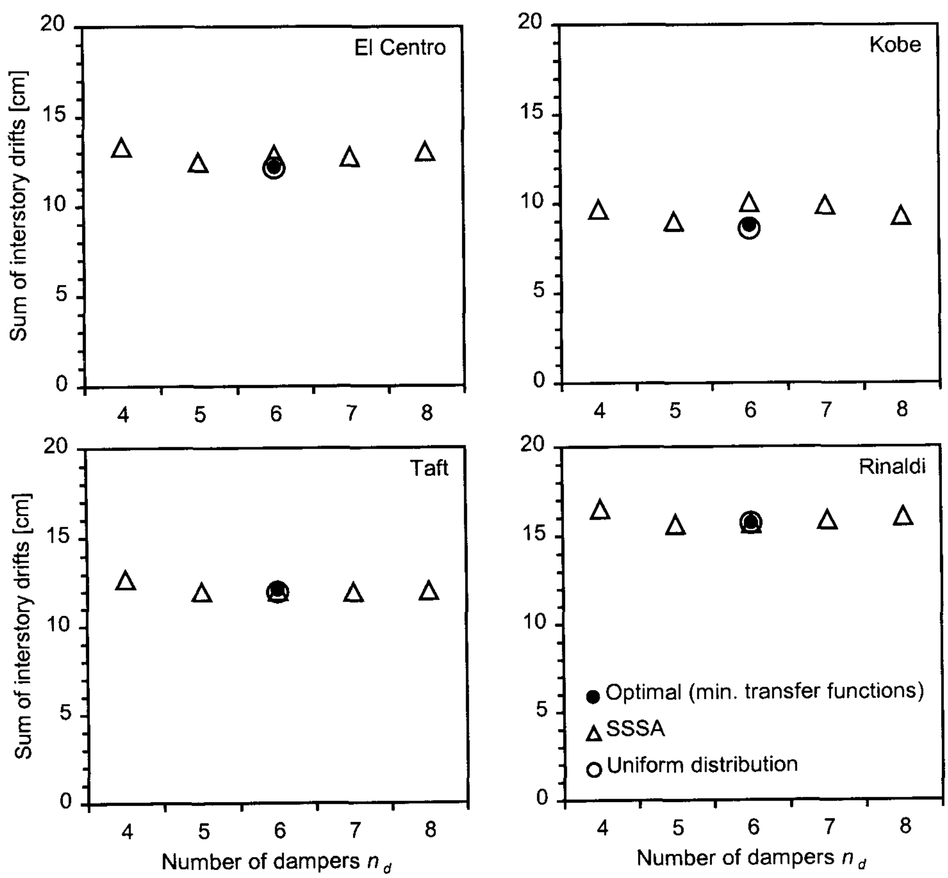


**Figure 4.** Comparison between the SSSA and optimal placement for minimum transfer functions: sum of interstory drifts for the six-story structure.

The story stiffnesses of the six-story structure shown in Figure 3 were chosen in such a way that amplitudes of interstory drift transfer functions are the same at all stories when dampers are placed according to what has previously defined as uniform distribution (Takewaki 1997). In other words, the parameters of the six-story structure are such that the uniform distribution of added dampers is very close to the optimal configuration. Therefore, in this particular and very special case, the fact that the sum of interstory drifts corresponding to damper configurations given by the SSSA is sometimes greater than that given by the uniform distribution does not indicate that the sequential approach fails in optimizing damper locations. Actually, the fact that in this case damper configurations given by the SSSA tend to be similar to the uniform distribution (Table 3) indicates further that the sequential approach indeed leads to optimal damper arrangements.

## APPLICATION OF THE SSSA TO OTHER KINDS OF DAMPERS

With proper modifications, the SSSA can be applied for any kind of linear passive energy dissipation devices. First, factors $\alpha_1$ and $\alpha_2$ should be chosen in such a way that the resulting optimal location index (Equation 1) indeed indicates location at which the effect of the added damper is maximized. Second, appropriate equations to assess the damper size (i.e., Equation 7 for linear viscous dampers) should be developed in accordance with the mechanical behavior of the dampers to be used.

For instance, for the case of viscoelastic dampers, the SSSA may be applied as follows. Factor $\alpha_1$ and $\alpha_2$ should be given by:

$$\alpha_1 = \frac{1}{1+\eta} \quad , \quad \alpha_2 = \frac{\eta}{\omega\,(1+\eta)} \tag{12}$$

where $\eta$ is the loss factor of the viscoelastic material and $\omega$ is the natural circular frequency of structure *with* the supplemental viscoelastic dampers. Equation 12 was derived assuming harmonic motion at frequency $\omega$. Damper sizes can be estimated using the equations provided by Zhang and Soong (1992).

While the SSSA is expected to lead to efficient damper configurations for any kind of linear devices, there is still a question about the effectiveness of the SSSA when applied to nonlinear devices such as hysteretic dampers. In the case of friction dampers, comparison between the efficiency of the SSSA and the efficiency of other procedures is the objective of an ongoing investigation. It must be said that there is already a method for the design of friction dampers (Filiatrault and Cherry 1990) that is practical as defined before.

## CONCLUDING REMARKS

The Simplified Sequential Search Algorithm proposed in this paper for optimization of damper configurations is simple and practical. Although not entirely conclusive, the evidence included in this paper shows that the efficiency of the SSSA is at least comparable with the efficiency of other, more sophisticated methods, which are neither simple nor practical.

There are, however, two limitations of the SSSA that are worth mentioning. Firstly, resulting damper configurations depend on the ground motion used as seismic excitation in the sequential procedure. Although it was found that resulting damper configurations for the structures considered in this study were similar for different ground motions, this may not be always the case. Therefore, ground acceleration time histories to be used should be carefully characterized. Secondly, the usefulness of the SSSA is limited to those cases where the response of the structure with added dampers remains within the linear range.

## ACKNOWLEDGMENTS

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## REFERENCES CITED

Cheng, F. Y. and Pantelides, C. P., 1988, Optimal placement of actuators for structural control, *Technical Report NCEER-88-0037*, National Center for Earthquake Engineering Research, Buffalo, NY.

Computers and Structures Inc., 2000, *SAP-2000: Static and Dynamic Finite Element Analysis of Structures (Nonlinear Version 7.40)*, Computers and Structures Inc., Berkeley, CA.

Constantinou, M. C. and Symans, M. D., 1992, Experimental and analytical investigation of seismic response of structures with supplemental fluid viscous dampers, *Technical Report NCEER-92-0032*, National Center for Earthquake Engineering Research, Buffalo, NY.

Constantinou, M. C., Soong, T. T., and Dargush, G. F., 1998, *Passive energy dissipation systems for structural design and retrofit*, Monograph No. 1, Multidisciplinary Center for Earthquake Engineering Research, Buffalo, NY.

Filiatrault, A. and Cherry, S., 1990, Seismic design spectra for friction-damped structures, *Journal of Structural Engineering*, ASCE, **116** (5), 1334-1355.

Gluck, N., Reinhorn, A. M., Gluck, J., and Levy, R., 1996, Design of supplemental dampers for control of structures, *Journal of Structural Engineering*, ASCE, **122** (12), 1394-1399.

Reinhorn, A. M., Li, C., and Constantinou, M. C., 1995, Experimental and analytical investigation of seismic retrofit of structures with supplemental damping, Part 1: Fluid viscous damping devices, *Technical Report NCEER-95-0001*, National Center for Earthquake Engineering Research, Buffalo, NY.

Ribakov, Y. and Gluck, J., 1999, Optimal design of ADAS damped MDOF structures, *Earthquake Spectra*, **15** (2), 317-330.

Seleemah, A. A. and Constantinou, M. C., 1997, Investigation of seismic response of buildings with linear and nonlinear fluid viscous dampers, *Technical Report NCEER-97-0004*, National Center for Earthquake Engineering Research, Buffalo, NY.

Soong, T. T. and Dargush, G. F., 1997, *Passive Energy Dissipation Systems in Structural Engineering*, John Wiley and Sons Ltd., London (UK) and New York (USA).

Shukla, A. K. and Datta, T. K., 1999, Optimal use of viscoelastic dampers in building frames for seismic force, *Journal of Structural Engineering*, ASCE, **125** (4), 401-409.

Takewaki, I., 1997, Optimal damper placement for minimum transfer functions, *Earthquake Engineering and Structural Dynamics*, **26** (11), 1113-1124.

Zhang, R-H. and Soong, T. T., 1992, Seismic design of viscoelastic dampers for structural applications, *Journal of Structural Engineering*, ASCE, **118** (5), 1375-1392.